Didactic-mathematical knowledge of future mathematics teachers in Ecuador when developing tasks based on ethnomathematical practices

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**Abstract**

This paper aims to infer characteristics of the didactic-mathematical knowledge of future mathematics teachers when developing tasks based on local ethnomathematical practices. Ethnomathematical practices and mathematical knowledge of these future teachers of an
Ecuadorian University were identified and later their reflections on bringing these practices to the classroom and their justifications were analyzed. The results indicate positive contemplation by the future teachers on two counts: first, in connecting ethnomathematical practices with local professional activities and second, in presenting fragilities in the characteristics of both mathematical and didactic-mathematical knowledge. As a way forward, the paper suggests that these insights are relevant to be incorporated in future teachers’ training in addition to the didactic-mathematical knowledge and perspectives of Ethnomathematics.

**Keywords:** Didactic-mathematical knowledge. Ontosemiotic approach. Ethnomathematical practices. Teacher training

**Conocimiento didáctico-matemático de futuros profesores de matemáticas del Ecuador al desarrollar tareas basadas en prácticas etnomatemáticas**

**Resumen**
Este trabajo pretende inferir características del conocimiento didáctico-matemático de futuros profesores de matemáticas al desarrollar tareas basadas en prácticas etnomatemáticas locales. Se identificaron las prácticas etnomatemáticas y los conocimientos matemáticos de estos futuros profesores, estudiantes de una universidad ecuatoriana y, posteriormente, se analizaron sus reflexiones con relación al llevar estas prácticas al aula de clases. Los resultados indican una contemplación positiva por parte de los futuros profesores en dos aspectos: primero, en conectar las prácticas etnomatemáticas con las actividades profesionales locales y segundo, en presentar fragilidades en las características del conocimiento tanto matemático como didáctico-matemático. Como camino a seguir, el artículo sugiere que estas percepciones deben incorporarse en la formación de los futuros profesores, además de los conocimientos didáctico-matemáticos y las perspectivas de las Etnomatemáticas.

**Palabras clave:** Conocimiento didáctico-matemático. Enfoque Ontosemiótico. Prácticas etnomatemáticas. Formación de profesores.

**Conhecimento didático-matemático de futuros professores de matemática do Equador ao desenvolver tarefas baseadas em práticas etnomatemáticas.**

**Resumo**
Este trabalho visa inferir características do conhecimento didático-matemático de futuros professores de matemática ao desenvolver tarefas baseadas em práticas etnomatemáticas locais. As práticas etnomatemáticas e o conhecimento matemático destes futuros professores, estudantes de uma universidade equatoriana, foram identificados e, posteriormente, suas reflexões para levar estas práticas para a sala de aula foram analisadas. Os resultados indicam uma contemplação positiva por parte dos futuros professores em dois aspectos: primeiro, na conexão das práticas etnomatemáticas com as atividades profissionais locais e, segundo, na apresentação de fragilidades relacionadas às características do conhecimento matemático e didático-matemático. Como um caminho a seguir, o artigo sugere que estas percepções são relevantes para serem incorporados na formação de futuros professores, além dos conhecimentos didático-matemáticos e a perspectiva da Etnomatemática.

**Palavras-chave:** Conhecimento didático-matemático. Abordagem Ontossemiótica. Práticas etnomatemáticas. Formação de professores.
Introduction

Ethnomathematics can be understood from different perspectives (BREDA; DO ROSÁRIO LIMA, 2011). One of them is to consider it as the set of modes, styles, arts and techniques to explain, learn, and know, the natural, social, cultural and imaginary environments of a certain cultural group, (D'AMBRÓSIO, 2014)

From this conceptualization, Ethnomathematics can be considered as a research program in the search for an educational action, which, according to D'Ambrósio (1998) came to combat the traditional methods of both teaching and production of scientific knowledge, valuing, in this way, the different knowledge and techniques of and in the different sociocultural environments, conceptualizing mathematics as a cultural and social product (GERDES, 1991; KNIJNIK, 1996; ROSA, 2005); that is, it is a program that aims to consider the culture and more specifically, interculturality, as a space for the learning of mathematics, looking for ways to understand the meaning of mathematical objects, immersed in the context. This approach calls into question the importance of research practice in Ethnomathematics by teachers, showing, according to Domite (2004), how this trend in mathematics education influences the transformation of the teacher and the future teacher and their knowledge (BREDA; DO ROSÁRIO LIMA; GUIMARÃES, 2012).

Another theoretical approach interested both in the analysis of mathematical activity, considered as a historical-social and historical-cultural practice, and in the knowledge that the teacher must have in order to teach it, is the Ontosemiotic Approach (OSA, from now on) (GODINO; BATANERO; FONT, 2007, 2019). This approach offers us theoretical-methodological tools that help us describe and explain the teaching and learning processes of mathematics and, also, assess them as suitable or adequate. In particular, in this framework, there is a model of Didactic-Mathematical Knowledge (DMK) which, in Latin terms, refers to teachers' knowledge about mathematical content (MK) and their teaching (DK). This model interprets and characterizes the teacher's knowledge from three dimensions: mathematical dimension, didactic dimension and meta-didactic-mathematical dimension (PINO-FAN; GODINO, 2015). The first two allow the teacher to describe and explain the teaching and learning processes of mathematics, while the didactic-mathematical meta dimension is the one that helps us to assess the processes of instruction from the use of the tool Didactic Suitability
Criteria (DSC, from now on) — epistemic, cognitive, interactional, affective, mediational and ecological (BREDA; FONT; PINO-FAN, 2018; GODINO, 2013).

Oliveras and Godino (2015) carry out a theoretical exercise of comparison and articulation between the Ethnomathematics and the OSA presenting the basic characteristics of both theoretical frameworks, comparing the paradigmatic issues addressed and identifying concordances and complementarities between them. Along similar lines, Garcia and Rodríguez-Nieto (2022), analyze the use of unconventional measures in Mexican textbooks considering the ethnomathematical perspective and OSA, in addition to studies related to mathematical connections (RODRÍGUEZ-NIETO, 2020, 2021). Also, one can observe the studies of Fernández-Oliveras, Blanco-Álvarez and Oliveras (2022), Blanco-Álvarez, Fernandez-Oliveras and Oliveras (2017a, 2017b), which, from an adaptation of the DSC of the OSA for a teaching proposal in the perspective of Ethnomathematics, analyze the design and application of a proposal of the teaching of unconventional measurement patterns with primary school students in Colombia.

In the field of teacher training, Blanco-Álvarez (2017) has identified elements for the design of mathematics teacher training programs from an ethnomathematical perspective. From an articulation amongst Ethnomathematics, Philosophy of Language, the Ontosemiotic Approach to Knowledge and Mathematical Instruction, he concludes that there are necessary elements when designing a teacher training program from Ethnomathematics, which are: internal to the classroom and related to the human subjects protagonists of learning and teaching; internal to the classroom and related to discourse mediators, such as resources, institutional norms and curriculum; external to the classroom and related to the education system; and external to the classroom and related to the social system.

Ethnomathematics is perceived as a current trend in the teaching and learning of mathematics, has promoted a curricular reconceptualization in some Latin American countries (e.g. Ecuador) (ROSA, 2005) and has been explicitly considered, as a subject present in some curricula of initial training programs for mathematics teachers (e.g. of Ecuadorian public universities). This perspective combines with the Didactic Suitability Criteria, supports the future teacher in the assessment of didactic proposals for the learning of mathematics.

Although there is research that tries to articulate Ethnomathematics and OSA, there is no research that is concerned with investigating the didactic-mathematical knowledge of future
teachers when working or developing tasks from the perspective of ethnomathematical practices. Considering the obligatory curricular nature and the importance of working on Ethnomathematics in the initial training of mathematics teachers, this research aims to infer characteristics of the didactic-mathematical knowledge of future mathematics teachers in Ecuador when developing tasks based on local ethnomathematical practices.

1. Theoretical framework

In this section, we will explain aspects related to Ethnomathematics, teacher training and, also, the DMK model of the OSA.

1.1 Ethnomathematics and teacher training

In a study conducted by Breda (2011) Breda and Do Rosário Lima (2011), a literature review was revealed that relates to the Ethnomathematics program and the training of mathematics teachers. In that subsection, we present this review in summary form.

According to D’Ambrósio (2014), to work from the perspective of Ethnomathematics in the school space is to contribute to the new generations to know and recognize much more cultural mathematics, linked to the daily life of various ethnic groups (D’AMBRÓSIO, 2008). It is a didactic position that seeks an improvement in the teaching-learning process of the discipline with the incorporation into the mathematical curriculum of knowledge derived from the student's life and human values, such as, for example, cooperation, solidarity and ethics. In addition, D’Ambrósio, (1998) affirms that ethnomathematical practices are born of research, which is why it is considered a research program and tends to become a proposal for educational action, where the role of the teacher is fundamental since it is he who is closing the gap between research and education.

According to Gerdes (1996, p.126), teacher training should include preparation for them to "investigate the ideas and practices of their cultural communities, ethnic and linguistic backgrounds and to look for ways to build their teaching around them [...] and contribute to mutual understanding, respect and appreciation of (sub) cultures and activities." Therefore, it is thought that, according to Moreira (2004), the perspective of Ethnomathematics on teacher training and professional development puts as a central theme the importance of acquiring theoretical-methodological tools capable of helping teachers to understand and pedagogically
appropriate mathematical diversity, in the communities where they teach, to integrate them into teaching and organize their practice, developing didactic activities that include mathematical elements from different cultural backgrounds.

In that sense, for Domite (2004), Ethnomathematics is integrated as a confluence between the personal and the professional life of the teacher, in which the central point is the group to be investigated. However, the teacher's stance should be hesitant in the sense of leaving some questions open for reflection. After all, who is the group to investigate? What logic does the teacher use to express his knowledge? For this, it is necessary to place as a point of reference the context and the place where one works and also to contemplate the modes of communication present in that particular place. According to the same author (2004), the research professor of ethnomathematical practices lives his research in a process of surprise and some tension, because, in effect, an analysis of certain ways of explaining and knowing in a certain group leads the researcher to a process of elaboration of new meanings that imply a flight of mathematics as a discipline, and thus it allows us to work on the articulation of other areas, such as history and economics, among others.

1.2 Didactic-mathematical knowledge model

In the Didactic-Mathematical Knowledge Model (DMK) of OSA, according to Pino-Fan and Godino (2015), the mathematical dimension of the DMK includes two subcategories of knowledge: common knowledge of the content (knowledge, about a specific mathematical object that is considered sufficient to solve problems or tasks proposed in the mathematics curriculum of a given educational level) and expanded knowledge of the content (it is further on in the curriculum of the educational level in question, or at a next level).

On the other hand, the didactic dimension of DMK includes the following subcategories of knowledge: specialized knowledge of the mathematical dimension (epistemic facet); knowledge about the cognitive aspects of students (cognitive facet); knowledge about the affective, emotional and attitudinal aspects of students (affective facet); knowledge about the interactions that arise in the classroom (interactional facet); knowledge about resources and means that can enhance student learning (mediational facet); and knowledge about the curricular, contextual, social, political, economic aspects, which influence the management of student learning (ecological facet).
Finally, the meta-didactic-mathematical dimension characterizes the knowledge that teachers need to reflect on their practice, identify and analyze the set of norms and meta-norms that regulate the teaching and learning processes of mathematics, and evaluate the didactic suitability to find possible improvements in the design and implementation of these processes (BREDA; PINO-FAN; FONT, 2017; PINO-FAN; GODINO; FONT, 2016).

For each of the components of didactic-mathematical knowledge, the OSA has "theoretical and methodological" tools that have been described and used in several investigations (GODINO, 2009, 2012; GODINO; BATANERO; FONT, 2019).

For instance, for the development of instruments that allow to evaluation and systematic analyze the knowledge of teachers regarding the mathematical dimension (common and expanded knowledge) and the epistemic facet of DMK, there is the tool "ontosemiotic configuration". This tool allows to describe and characterize the primary mathematical objects – representations/language (terms, expressions, notations, graphs) in their diverse registers; problem situations (intra- or extra-mathematical applications, exercises); concepts and definitions (introduced through definitions or descriptions); propositions (statements about concepts); procedures (algorithms, operations, calculation techniques); and arguments/justifications (statements used to validate or explain propositions or procedures) - that are produced through the respective mathematical processes of communication, problematization, definition, enunciation, elaboration of procedures (creation of algorithms and routines) and argumentation that are activated in the mathematical practices that teachers develop in solving a problem (MALASPINA; FONT, 2010), or as part of a problem planning (or problem sequence) for the classroom (MALASPINA, 2017; MALASPINA; TORRES; RUBIO, 2019; TORRES, 2020). In addition, mathematical knowledge includes the description of errors and ambiguities committed by teachers from a mathematical point of view (SÁNCHEZ et al., 2021).

For the development of instruments that allow to systematically evaluate and analyze the knowledge of teachers regarding the meta-didactic-mathematical dimension, the Didactic Suitability Criteria (DSC) tool is operational (BREDA; FONT; PINO-FAN, 2018). According to Font, Planas and Godino (2010), the DSC is characterized as follows: Epistemic Suitability, to assess if the mathematics being taught is "good mathematics"; Cognitive Suitability, to assess, before starting the instructional process, if what one wants to teach is at a reasonable distance
from what students know, and after the process, if the acquired learning is close to what was intended to be taught; *Interactional Suitability*, to assess if interactions resolve students’ doubts and difficulties; *Mediational Suitability*, to assess the adequacy of the material and temporal resources used in the instructional process; *Affective Suitability*, to assess the involvement (interests and motivations) of students during the instructional process; *Ecological Suitability*, to assess the adequacy of the instructional process to the educational project of the school or institution, the curricular guidelines, the conditions of the social and professional environment.

These criteria are split into components and indicators to become operational in the exercise of analysis and assessment of instructional processes. The criteria and components of didactic suitability are detailed in Table 1. The full table with the indicators can be found in Breda, Pino-Fan and Font (2017).

**Table 1 – Didactic suitability criteria and components**

<table>
<thead>
<tr>
<th>Didactic Suitability Criteria (DSC)</th>
<th>Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>Epistemic</td>
<td>✓ Errors, ambiguities, richness of processes, representativeness of the complexity of the mathematical object (problem-situations, representations, procedures, arguments, etc.)</td>
</tr>
<tr>
<td>Cognitive</td>
<td>✓ Prior knowledge, curricular adaptation to individual differences, learning, high cognitive demand</td>
</tr>
<tr>
<td>Interactional</td>
<td>✓ Teacher–student interaction, students’ interaction, autonomy, formative assessment</td>
</tr>
<tr>
<td>Mediational</td>
<td>✓ Material resources, number of students, class schedule and conditions, time</td>
</tr>
<tr>
<td>Affective</td>
<td>✓ Interests and needs, attitudes, emotions</td>
</tr>
<tr>
<td>Ecological</td>
<td>✓ Curriculum adaptation, intra- and interdisciplinary connections, social and labor usefulness, didactic innovation</td>
</tr>
</tbody>
</table>


Both the components and the indicators of the DSC have been made from a consensus present in the field of the educational community, considering the current trends in the teaching of mathematics, the principles and standards for the teaching of mathematics and the results of research in the area of Didactics of Mathematics. (BREDA; FONT; PINO-FAN, 2018; NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS [NCTM], 2000, 2014).

In particular, for epistemic suitability, a fundamental principle of OSA has been considered which, with the nuances of each approach, is (or can be) assumed by other theoretical approaches in the area. We refer to the principle that can be formulated as follows: mathematical objects emerge from practices, which entails their complexity. From this principle derives a
component (representativeness of the complexity of the notion to be taught) whose objective is to consider, as far as possible, this complexity in the design and redesign of the didactic sequences (FONT; GODINO; GALLARDO, 2013; FONT; PINO-FAN; BREDA, 2020; RONDERO; FONT, 2015; PINO-FAN et al., 2013).

The mathematical, didactic-mathematical and mathematical meta-didactic knowledge of the teacher, from the perspective of ethnomathematics, allows contemplating an extension of the epistemic suitability of the DMK, in particular, of the component "Representativeness of the complexity of the mathematical object to be taught". Fernández-Oliveras, Blanco-Álvarez and Oliveras (2022), propose an adaptation when working from the perspective of Ethnomathematics, see Table 2:

<table>
<thead>
<tr>
<th>Epistemic suitability component</th>
<th>Categories</th>
<th>Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Representativeness of the complexity of the mathematical object to be taught</td>
<td>Philosophical nature or stance</td>
<td>✓ Mathematics is referred to as a cultural product;</td>
</tr>
<tr>
<td></td>
<td>Problem Situations</td>
<td>✓ Extracurricular or ethnomathematical mathematical objects (contents) are made explicit in problem situations; problem situations are solved using different procedures, school and extracurricular algorithms;</td>
</tr>
<tr>
<td></td>
<td>Definitions, propositions, procedures, arguments</td>
<td>✓ Procedures, definitions, representations of extracurricular mathematical objects are presented; Arguments based on logics other than Western ones are valued and respected.</td>
</tr>
<tr>
<td></td>
<td>Relationships</td>
<td>✓ Comparisons, relationships between procedures, definitions, representations of school and extracurricular mathematical objects are established.</td>
</tr>
</tbody>
</table>

Source: (FERNÁNDEZ-OLIVERAS; BLANCO-ÁLVAREZ; OLIVERAS, 2022).

The notion of Didactic Suitability has had a relevant impact on teacher training in several countries, such impact evidence of the use of DSC in several investigations on teacher training who teach mathematics (BREDA; FONT; DO ROSÁRIO LIMA, 2015). But it is not used within the framework of a training device expressly designed to teach didactic suitability as a tool to organize reflection of teachers (BREDA, 2020; MORALES-LÓPEZ; FONT, 2019; MOREIRA; GUSMÃO; FONT, 2018; SALA-SEBASTIÀ et al., 2023; SALA-SEBASTIÀ; BREDA; FARSANI, 2022). However, there are already investigations in which it has been tried to promote the meta-didactic-mathematical knowledge of the teacher. In particular, some...
investigations were concerned with developing said knowledge in Ecuadorian mathematics teachers from the point of view of the "Representativeness of the complexity of the mathematical object to be taught" of epistemic suitability (CALLE, 2023; CALLE; BREDA; FONT, 2021, 2023).

2. Methodology

In this section we explain the context in which the study was developed, the data collection process and its respective analysis.

2.1 Background

The participants of this study were 48 future professors of Mathematics who study at a state university in southern Ecuador, who have studied the subject of Ethnomathematics—that corresponds to the fifth year of the Ordinary Academic Program (OAP)—in the second half of the year of 2019 and the beginning of the year of 2020 (before the Covid-19 pandemic).

The subject of Ethnomathematics, corresponding to the field of training integration of knowledge and contexts, is presented as a substantial contribution to the initial training of teachers of the Grade of Pedagogy of Experimental Sciences, once the design, planning, execution and evaluation of learning proposals are carried out, considering interculturality.

Through this subject, it is intended that the student manages to recognize and identify mathematics in the cultural activities of their environment, where they will demonstrate that mathematical knowledge, even if it is not school, is present in our cultural identity. Methodological strategies such as bibliographic review, concept maps, and proposals for collaborative work, have the possibility of making visible the characteristics of ethnomathematics and proposing, in a creative way, ways of understanding the meaning of mathematical objects, immersed in this context, with the support of formative research and through assisted teaching, experimentation and autonomous work as components of learning.

The subject begins with the historical review of what the Ethnomathematics program represents and its importance in culture of people, to continue with the analysis of the Ethnomathematics program in the Ecuadorian curriculum and complete with proposals for learning mathematics, through culture, through integrative projects coordinated by the Integrative Chair that guide the activities proposed by students. In particular, the subject aims to develop the ability to identify the mathematics present in the different expressions of our
culture, in order to design innovative educational proposals that support the solution to problems of the context. In this sense, the task that the 48 future teachers, organized into 11 working groups (WG1, WG2, WG3, WG4, WG5, WG 6, WG7, WG8, WG9, WG10, WG11), should carry out had the following objective: to investigate ethnomathematical practices of different social groups: jewelers, carpenters, cooks, tinsmiths, sculptors, etc. Explain how these practices would work in educational institutions. To do this, students should follow the following itinerary:

1. Choose a topic based on their cultural reality and that serves as a stage to promote students' interest in mathematics;
2. Attend and see the work of these social or cultural groups and the development of the work they do;
3. Identify mathematical processes in this practice;
4. Film or photograph the process, asking pertinent and necessary questions to prepare the corresponding report;
5. Expose the reflection on the experience, indicating if they would take it to the classrooms.

After following this pathway, the working groups should submit a final report (FR), which consisted of the following steps:

1. Description of the work performed;
2. Problem situations that arise and how to solve;
3. The procedures used by cultural groups, in their daily practices;
4. The language used by cultural groups;
5. Reflection on the practices and ethnomathematics, looking for connections with other areas such as history, anthropology, etc., considering the community;
6. Conclusions on the importance of these works and suggestions for application in teaching practice;
7. Presentation of the final report for the large class group.

2.2 Data collection and analysis

First, to identify and classify the ethnomathematical practices considered in the final reports made by the 11 WGs, we have worked with content analysis with emerging categories.
That is, since the emergence of the data, a classification of the ethnomathematical practices contemplated in the final reports made by the WGs has been made.

Secondly, in order to identify aspects of mathematical knowledge presented by future teachers, the final report (FR) described in the previous subsection has been considered, in particular stages 1, 2, 3 and 4 of the FR, in which some primary mathematical objects could emerge. To detect characteristics of mathematical knowledge, the notion of ontosemiotic configuration is present in the DMK model of the OSA (PINO-FAN; GODINO, 2015). Specifically, we sought to identify which primary mathematical objects - representations/languages, concepts and definitions, propositions, procedures and arguments/justifications - emerged in the mathematical practices contemplated in the works from a perspective of Ethnomathematics, in particular, has taken into account the competent “Representativeness of the complexity of the mathematical object to be taught” of the epistemic suitability proposed in Fernández-Oliveras, Blanco-Álvarez and Oliveras (2022). To this end, specifically, those present in Table 2 have been used as previous categories of analysis.

Thirdly, to identify characteristics of didactic knowledge, stages 5 and 6 of the final report have been considered, and for this purpose, the categories present in the Didactic Suitability Criteria tool present in Table 1 were used.

Finally, from a triangulation of the analyses among the most expert authors in the use of the instruments, it was possible to infer aspects of the didactic-mathematical knowledge of future teachers when working on tasks based on ethnomathematical practices. That inference was triangulated with the opinion of an expert in the theoretical framework of EOS.

3. Results
In this section, we present the results of the study. First, we show the emerging categories of classification of ethnomathematical practices considered in the work carried out by the 11 WGs formed by the 48 future teachers. Secondly, we show characteristics of the mathematical knowledge of these future teachers, having as a basis the primary objects of the OSA and the component "Representativeness of the complexity of the mathematical object to be taught" of the Epistemic Suitability. Finally, we infer characteristics of didactic-mathematical knowledge from the analysis of didactic suitability.
3.1 Classification of ethnomathematical practices

As a first result, two major categories of ethnomathematical practices considered by the WGs have been identified. The first is related to professional activities based on craft practices, which are upholstery, basket making, blacksmithing, the elaboration of toquilla straw hats, carpentry, haberdashery, pottery and poultry. The second category is the activity of games with pyrotechnics, see Table 3.

<table>
<thead>
<tr>
<th>Category</th>
<th>Types of ethnomathematical practices</th>
<th>Working Groups (WGs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Professional activity based on Crafts</td>
<td>Upholstery</td>
<td>WG2</td>
</tr>
<tr>
<td></td>
<td>Preparation of baskets</td>
<td>WG3</td>
</tr>
<tr>
<td></td>
<td>Smithy</td>
<td>WG8</td>
</tr>
<tr>
<td></td>
<td>Manufacture of straw hats toquilla</td>
<td>WG9</td>
</tr>
<tr>
<td></td>
<td>Carpentry</td>
<td>WG4</td>
</tr>
<tr>
<td></td>
<td>Haberdashery</td>
<td>WG5</td>
</tr>
<tr>
<td></td>
<td>Pottery</td>
<td>WG6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>WG7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>WG11</td>
</tr>
<tr>
<td></td>
<td>Poultry</td>
<td>WG10</td>
</tr>
<tr>
<td>Games Activity</td>
<td>Pyrotechnics</td>
<td>WG1</td>
</tr>
</tbody>
</table>

Source: authors.

3.2 Mathematical knowledge from epistemic suitability

To identify the characteristics of the mathematical knowledge of future the teachers when investigating ethnomathematical practices, as a second result, from steps 1 to 4 of the FR carried out by the WGs —description of the work done; problem situations that arise and how to solve; the procedures used by cultural groups, in their daily practices and the language used by cultural groups—, the emergence of some primary objects has been identified (Table 4). For this, it has been based on the ontosemiotic configuration of the primary objects of the EOS conjugated with the adaptation made by Fernández-Oliveras, Blanco-Álvarez and Oliveras (2022) for the component "Representativeness of the complexity of the mathematical object to be taught” of the epistemic suitability from the perspective of the Ethnomathematics.
Table 4– Primary objects from the perspective of emerging mathematics in the final reports of the working groups

<table>
<thead>
<tr>
<th>Epistemic suitability (Primary objects)</th>
<th>Characterization</th>
<th>Working Groups (WGs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Philosophical nature or stance</td>
<td></td>
<td>WG1, WG2, WG3, WG4,</td>
</tr>
<tr>
<td></td>
<td>Mathematics is a socio-cultural product</td>
<td>WG5, WG6, WG7, WG8,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>WG9, WG10, WG11</td>
</tr>
<tr>
<td>Situation-problem</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How to model the movement of fireworks when turned on?</td>
<td>WG1</td>
<td></td>
</tr>
<tr>
<td>How to make a rug in the shape of a six-pointed star?</td>
<td>WG2</td>
<td></td>
</tr>
<tr>
<td>How to build a model in wood?</td>
<td>WG4</td>
<td></td>
</tr>
<tr>
<td>How do you know how much money you have to give in exchange, and how much to charge for your products?</td>
<td>WG5</td>
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<tr>
<td>How to make an iron lantern?</td>
<td>WG8</td>
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</tr>
<tr>
<td>What are the measures to consider for a toquilla straw hat? What are the most common mistakes made in the preparation of toquilla straw hats?</td>
<td>WG9</td>
<td></td>
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<td>WG11</td>
<td></td>
</tr>
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<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
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<tr>
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<td>WG7</td>
</tr>
<tr>
<td>Manufacture of toquilla straw hats related to the calculation of dimensions, straw drying time, amount of straw</td>
<td></td>
<td>WG9</td>
</tr>
<tr>
<td>Preparation of skirts related to the formulation of series, sequences and geometry</td>
<td></td>
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</tr>
</tbody>
</table>

**Source:** the authors.
Concerning the objects that emerge from ethnomathematical practices, there are the nature of mathematics, problem situations, processes, representations/languages, arguments/justifications and relationships. No definitions or propositions were identified. All WGs assume a philosophical position that mathematics is a socio-cultural product. Seven groups have expressed problem situations from the perspective of ethnomathematical practices (see Table 4). However, four of them have expressed problem situations that are related to problems of another nature, such as, for example, the health of the person working in the blacksmithing activity.

As a first problem situation, we have the noise made when moulding the material with which you work (such as metal, copper, aluminium, among others), because the way in which they mould these materials is by means of the determined blow in certain areas and cause noise pollution for both workers and people who live near the place. (WG8).

The procedures that have emerged were direct measurement, indirect measurement, arithmetic calculation, counting, ordering and comparing, establishing sequences and patterns, applying proportions, and modelling geometric shapes. Although different procedures have emerged present in the ethnomathematical practices studied by the WGs, few groups manage to present representativeness of procedures, for example, the WG2 is the only one that presents more than three procedures used in the professional activity of carpentry.

About the languages and representations have emerged the graphic, analytical, verbal-written and iconic types. However, few WGs use a diversity of languages to explain a given ethnomathematical practice. For example, WG1 considers graphical, analytical, and verbal-written languages and representations. The WG2 group, when studying the practice of making mats, has used verbal-written, iconic and analytical language, see to Figure 1.
Figure 1 – Different languages/representations in mat making

Circular sector;
\[ S = r \cdot \alpha; \]  
\[ S = \frac{20\pi}{3} \]

We raise the arithmetic progression;
\[ a = u + (n - 1) \cdot d; \]
\[ \frac{20\pi}{3} = 1 + 20 \cdot x; \left( \frac{20\pi}{3} \right) - 1 = 20 \cdot x; \ x = 1 \]

The line corresponding to each point will decrease to one cm respectively on both sides of the circular sector until it reaches the tip of each end of the star. This procedure should be repeated on the other five ends of the mat, including the center.

Source: Done by WG2.

On the other hand, some WGs proposed as a language the nomenclature given to objects, utensils and procedures that are used in the different ethnomathematical practices, but without this language referring to an ethnomathematical representation of the respective activity. For example, WG5 proposed as the language used in the market:

*Our friend Maria, in her day to day, uses several ways to call her customers, this for her is a way to gain customers and demonstrate her good treatment towards other people, in addition to her education and as advertising, among some of the ways to treat her customers, are: “my little juice”, “My sweetheart”, “who sought heart”, “Come to my king/queen”. With these expressions, Doña María has won the affection of her customers, because we can see that many people exclusively frequent her sales position. (WG5).*

In relation to the arguments and justifications, arguments are observed to justify the procedures used, arguments to justify measuring instruments used (most of the WGs) and arguments to justify geometric properties. No other types of arguments have been observed, for example, an argument that justifies the types of representations used.

Concerning the relationships, it is observed that six WGs made relationships between the ethnomathematical activity investigated, and procedures used in mathematics. For example, the relationships that have emerged were: the process of making a star-shaped mat related to arithmetic progression; the manufacture and sale of clay objects related to a simple rule of three, ratios and proportions and system of equations; the production of pottery related to design and geometric properties; the manufacture of *toquilla* straw hats related to the calculation of
dimensions, straw drying time, amount of straw; and the preparation of skirts related to the formulation of series, sequences and geometry. Although the aforementioned relationships have been identified, no relationships have been observed, for example, amongst definitions, relationships amongst languages, or amongst concepts.

3.3 Analysis of Didactic Suitability

To identify the characteristics of the didactic-mathematical knowledge of future teachers when investigating ethnomathematical practices, stages 5 and 6 of the IF - reflection on ethnomathematics —have been taken into account, looking for connections with other areas such as history, anthropology, etc., taking into account the community; conclusions on the importance of these works and suggestions for application in teaching practice— and DSC has been used as an analysis tool (Table 2).

Epistemic suitability

In addition to the analysis presented in the previous subsection, which has considered the knowledge of future teachers concerning the component "Representativeness of the complexity of the mathematical object to be taught" from the perspective of Ethnomathematics, about, to the epistemic suitability, it is observed that most of the WGs consider the importance of working different mathematical processes present in ethnomathematical practices in the school environment. For example, the process of extra mathematical connections, algorithmization, visualization/identification, estimation, trial and error, formulation of conjunctures, problem-solving process, experimentation and modelling. The following evidence shows the reflection of WG1 about the importance of working on the processes of visualization/identification and estimation. Next, evidence from WG2 is presented that reaffirms the importance of the process of extra-mathematical connection between cultural activity and mathematics worked on in school. Finally, one of the WP6 reflects on the processes of experimentation and modelling.

In the problems studied on the way and procedure of making castles and fireworks, we have observed their great importance and influence on cultural and religious traditions that in addition to the visual delight of the people who observe it serve as a sample to identify geometric shapes and figures, as well as the estimation of the space and volume required to develop these beautiful acts. (WG1)
We suggest that the fabrics, ceramics, etc. of our people be used in teaching practice to be worked from the mathematical point of view so that students relate to their environment the different ways of doing mathematics, guided by these clear examples that are found in our beloved city and even more in our country that is megadiverse. (WG2).

I believe that it would be very helpful to implement workspaces in educational institutions where students can learn to carry out activities that promote the growth of cultural identity, activities such as: the manipulation and modelling of objects with mud or clay (WG6).

Apart from the components "Representativeness of the complexity of the mathematical object to be taught" and "Richness of processes", future teachers do not comment in their work on errors or ambiguities that may be committed during the process of instruction of a task or sequence of tasks based on a certain ethnomathematical practice.

**Cognitive Suitability**

It was possible to identify the comment of a group on the importance of considering the previous knowledge of the students. Also, there was a comment on the constructivist and social learning of the students from a perspective of Ethnomathematics:

*The teacher needs to review his teaching practice daily and develop a pedagogical project that always values the knowledge and history of each student (WG4)*

*It is important to implement these assignments in class, as this way students can have a socio-constructivist perspective. (WG10)*

However, there are no comments related to the form of evaluation or to the curricular adaptation to individual differences. Perhaps, one of the reasons for the latter is that by assuming that the class will work on tasks based on the different ethnomathematical practices present in the Ecuadorian context, this diversity would already be contemplated.

**Interactional suitability**

No reflections have been made on the interaction between teacher and student, nor on the interaction amongst students, nor on autonomy and formative evaluation.

**Mediational suitability**

Concerning to the media, a group commented about different types of manipulative materials that can be worked from the perspective of ethnomathematical practices. Comments on classroom conditions and teaching time were not contemplated.
Our students would learn from their contextual culture, from other ways of teaching mathematics, quipus and taptana, as teaching materials would help a better understanding of the student (WG11)

Affective suitability

Only two working groups made explicit comments regarding the interests and needs of the students when working on tasks from the cultural perspective.

Formulate learning proposals based on daily living activities which involving the use of mathematics, generates the interest of children and young people and even better if they respond to the needs of the community such as the elaboration of baskets or vessels, which have a cultural value, legacy of the ancestors and that maintain their usefulness. (WG3)

As future teachers, we encourage ourselves to keep present these projects within the classroom where each of the features that identify us with the desire to generate greater interest from future generations is highlighted, keeping our cultural identity alive. (WG8)

Ecological suitability

On the one hand, all the WGs have considered that working in mathematics classes from the perspective of Ethnomathematics is a way of contemplating the Ecuadorian educational curriculum. On the other hand, there were no comments on intramathematical connections work. However, some groups argued that it is possible to establish interdisciplinary connections by working under this perspective. For example, WG1 reflects that a connection between ethnomathematical practice and physics can be established, and a second group (WG7) reflects on the connection between microeconomics and accounting.

When lighting a firework (cartridges filled with gunpowder, used as fireworks pyrotechnics), it describes a series of movements that are taught through physics within the classroom; That is why the importance of the calculations and location of the dust that will explode. (WG1)
Figure 2 – graphical representation of thrown fireworks.

Source: WG1

In addition, other disciplines such as accounting, microeconomics and production are evidenced in the trade and daily practices of Don José Encalada, concepts of profitability, costs (raw material, labour and CIF), utility and price, etc. They intervene in the practice of pottery as a business. (WG7)

Finally, only two groups commented that working on school mathematics from the perspective of Ethnomathematics leads to didactic innovation.

Creativity and innovation based on crafts are to be presented as projects in the different educational institutions (WG2).

To develop in the classroom an educational proposal that encourages students and teachers in the development of creativity, giving rise to both new and rich forms of learning (WG4).

4. Discussion and considerations

This study aimed at drawing didactic-mathematical knowledge of Ecuador’s prospective mathematics teachers by analyzing their tasks development from an ethnomathematical lens. As a first result, it is observed that, in terms of ethnomathematical practices, the WGs considered professional activities, some based on handicraft practices, and some games. However, no ethnomathematical practices have been observed outside these two categories, such as, for example, the ways of life of indigenous peoples who have their languages, customs, traditions and forms of social and political organization, such as the Kichwa, Shuar, Achuar, Waorani, Tsáchila, an aspect considered in the following research (TUMIALÁN BONILLA et al., 2018).

As a second result, fragility has been observed in the mathematical knowledge of the future teachers when performing tasks based on ethnomathematical practices. For example,
although all WGs have as their philosophical position that mathematics is a socio-cultural product, none of them has considered the concepts or definitions present in the ethnomathematical practices investigated. In addition, few WGs present a representative sample of both the languages and representations, as well as the procedures present in these practices. The problem situations contemplated are limited and the arguments refer only to the procedures used by professionals of a certain professional activity investigated. This result corroborates what was found in Sala-Sebastià et al. (2023) when analyzing the didactic-mathematical knowledge of future teachers of Early Childhood Education when solving and designing robotic problems.

As a third result, it is inferred that the future teachers show fragility when it comes to didactic-mathematical knowledge, since, when relating the ethnomathematical practices studies, they face difficulties to articulate and contemplate crucial aspects of didactic knowledge required to work in the context of schools.

Most of the WGs have considered the mathematical processes that could be worked on based on ethnomathematical practices in the school, an aspect of epistemic suitability and compliance with the Ecuadorian curriculum when working from an intercultural perspective (an aspect present in ecological suitability). However, they didn’t consider other aspects of classroom management. Furthermore, few WGs have considered appropriate material resources that relate to the different local socio-cultural activities, almost no WGs referred to the issue of student learning, nor the issue of affectivity or the issue of extra or intra mathematical connections, the latter aspect being explored in RODRÍGUEZ-NIETO (2020, 2021).

The reasons for these results can be related to two main aspects. The first is that, although Ethnomathematics is inserted as a curricular component in the initial training of teachers in Ecuador, it still needs to be better worked in such training. Teacher training in this perspective involves acquiring theoretical-methodological tools to understand mathematical diversity in the communities where it is taught and developing didactic activities that integrate mathematical elements from various social and cultural backgrounds (MOREIRA, 2004). The second aspect is that in teacher training programs it is essential to train future teachers to acquire mathematical and didactic mathematical knowledge for the teaching of mathematics, as has been done in Calle (2023).
Finally, it is evident to design and implement mathematics teacher training programs from the perspective of Ethnomathematics, as proposed by authors such as Oliveras and Gavarrete (2012), but which consider the elements internal to the classroom (human subjects involved in the teaching and learning processes, institutional rules and the curriculum) and also the elements external to the classroom (relating to the education system and the social and cultural system). (BLANCO-ÁLVAREZ., 2017).

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